The National Strategies
Secondary

Assessing Pupils' Progress

Focused assessment materials: Level 8

department for children, schools and families

QCA Qualifications and Curriculum Authority

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Focused assessment materials: Level 8

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Acknowledgement

The National Strategies are grateful for the many contributions from teachers, consultants and students that helped to make these materials possible. Particular thanks are due to colleagues from Gloucestershire Local Authority for their contributions.

These materials are based on the APP assessment criteria and organised in the National Curriculum levels. There is a set for each of levels 4 to 8.

The focused assessment materials include for each assessment criterion:

- **Examples of what pupils should know and be able to do** so teachers have a feel for how difficult the mathematics is intended to be. These are not activities or examples that will enable an accurate assessment of work at this level. To do this, you need a broad range of evidence drawn from day-to-day teaching over a period of time; this is exemplified in the Standards files, which are provided as part of the overall APP resources.
- Some **probing questions** for teacher to use with pupils in lessons to initiate dialogue to help secure their assessment judgement.

Numbers and the number system

Examples of what pupils should know and be able to do	Probing questions
Understand the equivalence between recurring decimals and fractions	
Distinguish between fractions with denominators that have only prime factors 2 and 5 (which are represented as terminating decimals), and other fractions (which are represented by terminating decimals). Decide which of the following fractions are equivalent to terminating decimals: $\frac{3}{5}$, $\frac{3}{11}$, $\frac{7}{30}$, $\frac{9}{22}$, $\frac{9}{20}$, $\frac{7}{16}$ Write $0.\dot{4}\dot{5}$ as a fraction in its simplest terms.	Write some fractions which terminate when converted to decimals. What do you notice about these fractions? What clues do you look for when deciding if a fraction terminates? $\frac{1}{3}$ is a recurring decimal. What other fractions related to one-third will also be recurring? Using the knowledge that $\frac{1}{3} = 0.\dot{3}$, how would you go about finding the decimal equivalents of $\frac{1}{6}$, $\frac{1}{30}$…? $\frac{1}{11} = 0.\dot{0}\dot{9}$. How do you use this fact to express $\frac{2}{11}$, $\frac{3}{11}$, …$\frac{12}{11}$ in decimal form? If you were to convert these decimals to fractions: 0.0454545….., 0.454545……, 4.545454……, 45.4545….. Which of these would be easy/difficult to convert? What makes them easy/difficult to convert? Can you use the fraction equivalents of $4.\dot{5}\dot{4}$ and $45.\dot{4}\dot{5}$ to prove the second is ten times greater than the first? Which of the following statements are true/false? • All terminating decimals can be written as fractions. • All recurring decimals can be written as fractions. • All numbers can be written as a fraction.

Calculating

<table>
<tr><th>Examples of what pupils should know and be able to do</th><th>Probing questions</th></tr>
<tr><td colspan="2">Use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change</td></tr>
<tr><td>Solve problems involving, e.g. compound interest and population growth using multiplicative methods.
Use a spreadsheet to solve problems such as:
• How long would it take to double your investment with an interest rate of 4% per annum?
• A ball bounces to ¾ of its previous height each bounce. It is dropped from 8m. How many bounces will there be before it bounces to approximately 1m above the ground?
Solve problems in other contexts, such as:
• Each side of a square is increased by 10%. By what percentage is the area increased?
• The length of a rectangle is increased by 15%. The width is decreased by 5%. By what percentage is the area changed?</td><td>Talk me through why this calculation will give the solution to this repeated proportional change problem.
How would the calculation be different if the proportional change was…?
What do you look for in a problem to decide the product that will give the correct answer?
How is compound interest different from simple interest?
Give pupils a set of problems involving repeated proportional changes and a set of calculations. Ask pupils to match the problems to the calculations.</td></tr>
</table>

Solve problems involving calculating with powers, roots and numbers expressed in standard form, checking for correct order of magnitude and using a calculator as appropriate

Use laws of indices in multiplication and division, e.g. to calculate:

$$\frac{4^3 \times 4^5}{(4^2)^3}$$

What is the value of *c* in the following question?

$48 \times 56 = 3 \times 7 \times 2^c$

Understand index notation with fractional powers, e.g. knowing that $n^{1/2} = \sqrt{n}$ and $n^{1/3} = \sqrt[3]{n}$ for any positive number *n*.

Convert numbers between ordinary and standard form, e.g.

$734.6 = 7.346 \times 10^2$

$0.0063 = 6.3 \times 10^{-3}$

Use standard form expressed in conventional notation and on a calculator display. Know how to enter numbers on a calculator in standard form.

Use standard form to make sensible estimates for calculations involving multiplication and division.

Solve problems involving standard form, such as:

Given the following dimensions

Diameter of the eye of a fly: 8×10^{-4}m

Height of a tall skyscraper: 4×10^2m

Height of a mountain: 8×10^3m

How many times taller is the mountain than the skyscraper?

How high is the skyscraper in km?

Convince me that:

$3^7 \times 3^2 = 3^9$

$3^7 \div 3^{-2} = 3^9$

$3^7 \times 3^{-2} = 3^5$

When working on multiplications and divisions involving indices, ask:

Which of these are easy to do? Which are difficult? What makes them difficult?

How would you go about making up a different question that has the same answer?

What does the index of ½ represent?

What are the key conventions when using standard form?

How do you go about expressing a very small number in standard form?

Are the following statements always, sometimes or never true?

- Cubing a number makes it bigger.
- The square of a number is always positive.
- You can find the square root of any number.
- You can find the cube root of any number.

If sometimes true, precisely when is the statement true and when is it false?

Which of the following statements are true?

$16^{3/2} = 8^2$

the length of an A4 piece of paper is 2.97×10^5km

$8^{-3} = 1/2^{-9}$

$27^2 = 3^6$

$3\sqrt{7} \times 2\sqrt{7} = 5\sqrt{7}$

Algebra

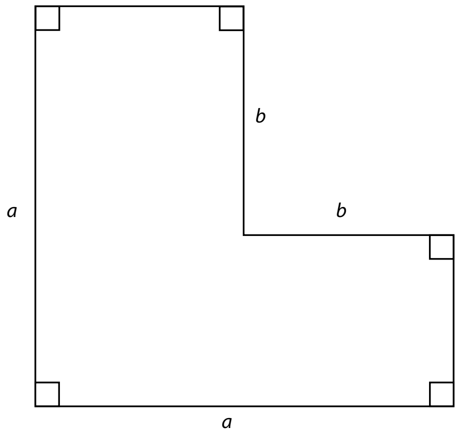
Examples of what pupils should know and be able to do	Probing questions
Factorise quadratic expressions including the difference of two squares	
Factorise: $x^2 + 5x + 6$ $x^2 + x - 6$ $x^2 + 5x$ Recognise that: $x^2 - 9 = (x + 3)(x - 3)$	When reading a quadratic expression that you need to factorise, what information is critical for working out the two linear factors? What difference does it make if the constant term is negative? What difference does it make if the constant term is zero? Talk me through the steps you take when factorising a quadratic expression. Show me an expression which can be written as the difference of two squares. How can you tell? Why must 1000×998 give the same result as $999^2 - 1$?
Manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions	
Solve linear equations involving compound algebraic fractions with positive integer denominators, e.g.: $\frac{(2x-6)}{4} - \frac{(7-x)}{2} = -6$ Simplify: $(2x - 3)(x - 2)$ $10 - (15 - x)$ $(3m - 2)^2 - (1 - 3m)^2$ Cancel common factors in a rational expression such as: $\frac{(x + 3)^2}{x + 3}$ Expand the following, giving your answer in the simplest form possible: $(2b - 3)^2$	Give pupils examples of the steps towards the solution of equations with typical mistakes in them. Ask them to pinpoint the mistakes and explain how to correct. Talk me through the steps involved in simplifying this expression. What tells you the order of the steps? How do you go about finding common factors in algebraic fractions? Give me three examples of algebraic fractions that can be cancelled and three that cannot be cancelled. How did you do it? How is the product of two linear expressions of the form $(2a \pm b)$ different from $(a \pm b)$?

Derive and use more complex formulae and change the subject of a formula

Change the subject of a formula, including cases where the subject occurs twice such as:

$y - a = 2(a + 7)$

By formulating the area of the shape below in different ways, show that the expressions $a^2 - b^2$ and $(a - b)(a + b)$ are equivalent.



Derive formulae such as:

- the area A of an annulus with outer radius r_1 and inner radius r_2:
  $A = \pi (r_1^2 - r_2^2)$
- the perimeter p of a semicircle with radius r:
  $p = r(\pi + 2)$

How do you decide on the steps you need to take to rearrange a formula? What are the important conventions?

What strategies do you use to rearrange a formula where the subject occurs twice?

Talk me through how you went about deriving this formula.

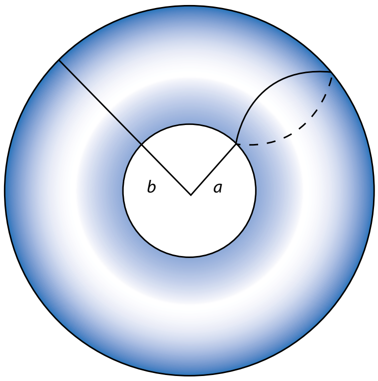
Evaluate algebraic formulae, substituting fractions, decimals and negative numbers

Substitute integers and fractions into formulae, including formulae with brackets, powers and roots, e.g.:

$p = s + t + (5\sqrt{(s^2 + t^2)})/3$

Work out the value of p when $s = 1.7$ and $t = 0.9$.

Clay is used to make this shape, a torus, with radii $a = 4.5$ and $b = 7.5$



Its volume is $\frac{1}{4}\pi^2 (a + b)(b - a)^2$
Work out the volume of clay used.

Given a list of formulae ask: If you are substituting a negative value for the variable, which of these might be tricky? Why?

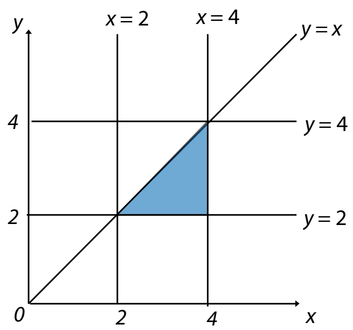
Talk me through the steps involved in evaluating this formula. What tells you the order of the steps?

From a given set of algebraic formulae, select the examples that you typically find easy/difficult.

What makes them easy/difficult?

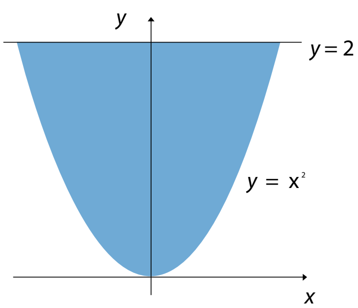
Solve inequalities in two variables and find the solution set

This pattern is formed by straight line graphs of equations in the first quadrant.



Write three inequalities to describe fully the shaded region.

The shaded region is bounded by the line $y = 2$ and the curve $y = x^2$.



What are the similarities and differences between solving a pair of simultaneous equations and solving inequalities in two variables?

Convince me that you need a minimum of three linear inequalities to describe a closed region.

How do you check if a point lies:

- inside the region
- outside the region
- on the boundary of the region?

Sketch, identify and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations

Match the shape of graphs to given relationships, e.g.:

- the circumference of a circle plotted against its diameter
- the area of a square plotted against its side length
- the height of fluid over time being poured into various shaped flasks.
- Interpret a range of graphs matching shapes to situations such as:
- the distance travelled by a car moving at constant speed, plotted against time
- the number of litres left in a fuel tank of a car moving at constant speed, plotted against time
- the number of dollars you can purchase for a given amount in pounds
- the temperature of a cup of tea left to cool to room temperature, plotted against time.

Identify how y will vary with x if a balance is arranged so that 3kg is placed at 4 units from the pivot on the left-hand side and balanced on the right-hand side by y kg placed x units from the pivot.

Show me an example of an equation of a quadratic curve which does not touch the x-axis. How do you know?

Show me an example of an equation of a parabola (quadratic curve) which:

- has line symmetry about the y-axis
- does not have line symmetry about the y-axis.

How do you know?

What can you tell about a graph from looking at its function?

Show me an example of a function that has a graph that is not continuous, i.e. cannot be drawn without taking your pencil off the paper. Why is it not continuous?

How would you go about finding the y value for a given x value? And an x value for a given y value?

Understand the effect on a graph of addition of (or multiplication by) a constant

Given the graph of $y = x^2$, use it to help sketch the graphs of $y = 3x^2$ and $y = x^2 + 3$.

Explore what happens to the graphs of the functions, e.g.:

- $y = ax^2 + b$ for different values of a and b
- $y = ax^3 + b$ for different values of a and b
- $y = (x \pm a)(x \pm b)$ for different values of a and b

Show me an example of an equation of a graph which moves (translates) the graph of $y=x^3$ vertically upwards (in the positive y-direction).

What is the same/different about: $y = x^2$, $y = 3x^2$, $y = 3x^2 + 4$ and $\frac{1}{3}x^2$?

Is the following statement always, sometimes or never true?

- As 'a' increases the graph of $y = ax^2$ becomes steeper.

Convince me that the graph of $y = 2x^2$ is a reflection of the graph of $y = -2x^2$ in the x-axis.

Shape, space and measures

Examples of what pupils should know and be able to do	Probing questions
Understand and use congruence and mathematical similarity	
Understand and use the preservation of the ratio of side lengths in problems involving similar shapes. Use congruent triangles to prove that the two base angles of an isosceles triangle are equal by drawing the perpendicular bisector of the base. Use congruence to prove that the diagonals of a rhombus bisect each other at right angles.	What do you look for when deciding whether two triangles are congruent? What do you look for when deciding whether two triangles are similar? Which of these statements are true? Explain your reasoning. • Any two right-angled triangles will be similar. • If you enlarge a shape you get two similar shapes. • All circles are similar. Convince me that: • any two regular polygons with the same number of sides are similar. • alternate angles are equal (using congruent triangles).

Understand and use trigonometrical relationships in right-angled triangles, and use these to solve problems, including those involving bearings

Use sine, cosine and tangent as ratios (link to similarity).

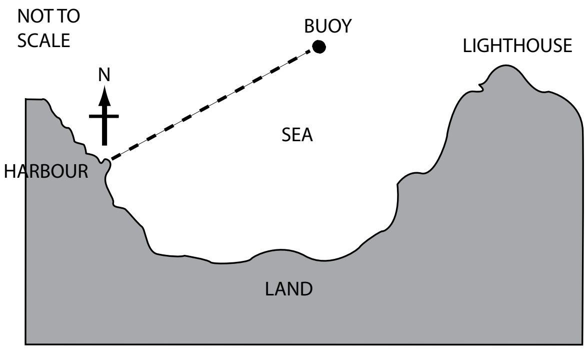
Find missing sides in problems involving right-angled triangles in two dimensions.

Find missing angles in problems involving right-angled triangles in two dimensions.

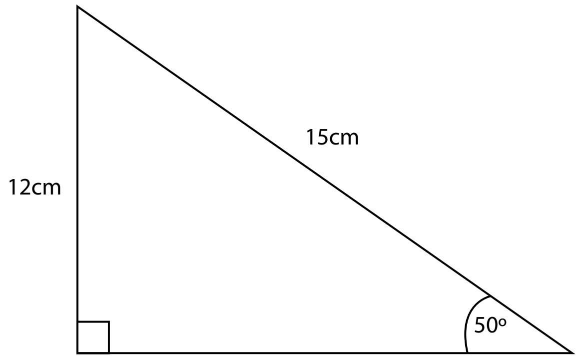
Sketch right-angled triangles for problems expressed in words.

Solve problems such as:

- Calculate the shortest distance between the buoy and the harbour and the bearing that the boat sails on. The boat sails in a straight line from the harbour to the buoy. The buoy is 6km to the east and 4km to the north of the harbour.



Is it possible to have a triangle with the angles and lengths shown below?



What do you look for when deciding whether a problem can be solved using trigonometry?

What's the minimum information you need about a triangle to be able to calculate all three sides and all three angles?

How do you decide whether a problem requires use of a trigonometric relationship (sine, cosine or tangent) or Pythagoras' theorem to solve it?

'You can use trigonometry to find missing lengths and/or angles in all triangles.' Is this statement true?

Why is it important to understand similar triangles when using trigonometric relationships (sine, cosines and tangents)?

Understand the difference between formulae for perimeter, area and volume in simple contexts by considering dimensions

Work with formulae for a range of 2-D and 3-D shapes and relate the results and dimensions.

How do you go about deciding whether a formula is for a perimeter, an area or a volume?

Why is it easy to distinguish between the formulae for the circumference and area of a circle?

How would you help someone to distinguish between the formula for the surface area of a cube and the volume of a cube?

How do you decide whether a number in a calculation represents the length of a dimension?

Handling data

<table>
<tr><th>Examples of what pupils should know and be able to do</th><th>Probing questions</th></tr>
<tr><td colspan="2">Estimate and find the median, quartiles and interquartile range for large data sets, including using a cumulative frequency diagram</td></tr>
<tr><td>Estimate and find the median, upper and lower quartiles and interquartile range for large data sets represented in:
a stem and leaf diagram
a cumulative frequency diagram.
Use a cumulative frequency diagram to answer questions, such as:
If 70% was the pass mark, how many passed the test?
Estimate the median and quartiles from a cumulative frequency diagram.</td><td>How would you go about making up a set of data with a median of 10 and an interquartile range of 7?
Convince me that the interquartile range for a set of data cannot be greater than the range.
How do you go about finding the interquartile range from a stem and leaf diagram? What is it about a stem and leaf diagram that helps to find the median and interquartile range?
How would you go about finding the median from a cumulative frequency table?
Why is it easier to find the median from a cumulative frequency graph than from a cumulative frequency table?</td></tr>
</table>

Compare two or more distributions and make inferences, using the shape of the distributions and measures of average and spread including median and quartiles

Construct, interpret and compare box plots for two sets of data, e.g. the heights of Year 7 and Year 9 pupils or the times in which Year 9 boys and Year 9 girls can run 100m.

Recognise positive and negative skew from the shape of distributions represented in:

- frequency diagrams
- cumulative frequency diagrams
- box plots.

What features of the distributions can you compare when using a box plot/a frequency diagram?

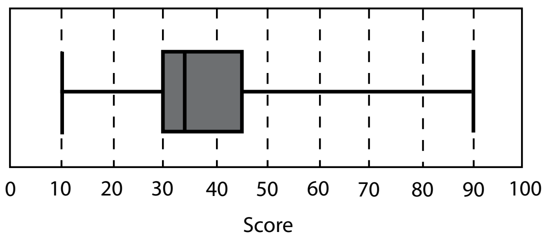
Make some comparative statements from the shape of each of these distributions.

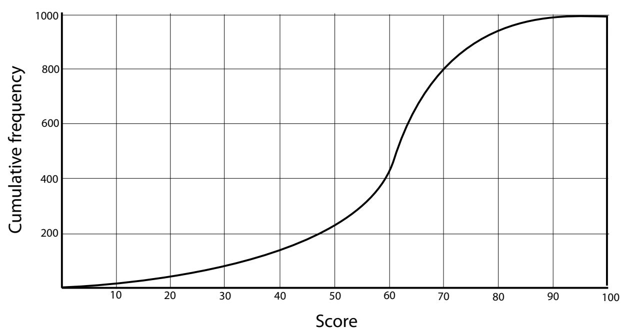
What are the key similarities and differences between these two distributions?

Describe two contexts, one in which a variable/attribute has negative skew and the other in which it has positive skew.

How can we tell from a box plot that the distribution has negative skew?

Convince me that the following representations are from different distributions, e.g.:





What would you expect to be the same/different about the two distributions representing, e.g.:

- heights of pupils in Years 1 to 6 and heights of pupils only in Year 6.

Know when to add or multiply two probabilities	
Recognise when probabilities can be associated with independent or mutually exclusive events. Understand that when *A* and *B* are mutually exclusive, then the probability of *A* or *B* occurring is $P(A) + P(B)$, whereas if *A* and *B* are independent events, the probability of *A* and *B* occurring is $P(A) \times P(B)$. Solve problems such as: • A pack of blue cards are numbered 1 to 10. What is the probability of picking a multiple of 3 or a multiple of 5? • There is also an identical pack of red cards. What is the probability of picking a red 5 and a blue 7?	Show me an example of: • a problem which could be solved by adding probabilities • a problem which could be solved by multiplying probabilities. What are all the mutually exclusive events for this situation? How do you know they are mutually exclusive? Why do you add the probabilities to find the probability of either this event or this event occurring? If I throw a coin and roll a dice the probability of a 5 and a head is $\frac{1}{12}$. This is not $\frac{1}{2} + \frac{1}{6}$. Why not? What are the mutually exclusive events in this problem? How would you use these to find the probability?
Use tree diagrams to calculate probabilities of combinations of independent events	
Construct tree diagrams for a sequence of events using given probabilities or theoretical probabilities. Use the tree diagram to calculate probabilities of combinations of independent events. The probability that it will rain on Tuesday is $\frac{1}{5}$. The probability that it will rain on Wednesday is $\frac{1}{3}$. What is the probability that it will rain on just one of the days? The probability that Nora fails her driving theory test on the first attempt is 0.1. The probability that she passes her practical test on the first attempt is 0.6. Complete a tree diagram based on this information and use it to find the probability that she passes both tests on the first attempt.	What are the key features of mutually exclusive and independent events on a tree diagram? Why do the probabilities on each set of branches have to sum to 1? How can you tell from a completed tree diagram whether the question specified with or without replacement? What strategies do you use to check the probabilities on your tree diagram are correct? Talk me through the steps you took to construct this tree diagram and use it to find the probability of this event.

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