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Bridge Building: A Practical Mathematics Task

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Introduction

The bridge building task is one that we have now used a number of times in different situations as part of the Centre for Teaching Mathematics' Mathematics Enrichment Programme. It has always proved both popular and challenging for students from year 8 to year 12. The actual task was inspired by the Dartford Bridge built in the early 1990's to provide an improved link across the River Thames. The practical task was an attempt to allow children to reproduce some aspects of the design of this type of bridge in a simple way. The students are required to build a model of this type of bridge using retort stands, cardboard bridge sections and thin elastic. A considerable amount of mathematics is used to determine the lengths of elastic prior to stretching. The elastic is then cut and the bridge assembled to test. This article describes the stages that are involved in this task and concludes with some comments about how students approach and solve the problem.

The practical importance of this type of bridge is that it uses a single support only (unlike, for example, suspension bridges) and that all the horizontal loads are contained within the structure, which makes it ideal for use when the river banks are fragile (as when they are alluvial mud), in contrast with suspension bridges which need a strong anchor for the cables or arch bridges which need a strong abutment at each end.

Outline

The bridge will consist of a deck consisting of a horizontal row of ballasted card boxes suspended from a central pair of vertical retort stands or similar towers, by means of pairs of thin elastic cords.

Building the bridge proceeds in stages. Firstly, the boxes are built, then the elastic is tested to see how its stretch varies with the angle to the vertical. Finally, the appropriate lengths of elastic are determined and the bridge is built. The whole task takes about 6 hours.

Building the Bridge

As not all elastic you purchase will behave in a linear elastic manner, teachers may find it useful to carry out the following activity.

Stage O-A teacher activity: Find the elastic characteristic of your elastic

Take about 35 cm of elastic. Fasten the elastic to a 1 metre ruler and attach a light carrier to the lower end so the length to be loaded is 30 cm. Add l0g masses to the carrier and measure the length for each load. Plot the length against the load. The result will be similar to Figure 1 below. The section where the gradient is greatest and almost linear is the part which is best to use.

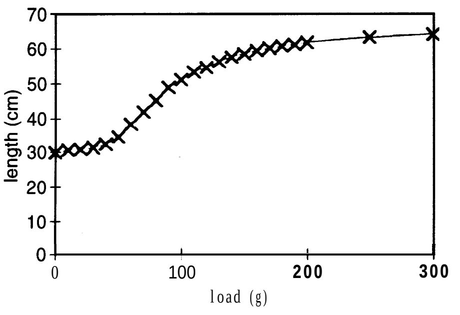

Fig. 1

For the best results, choose a total weight for the boxes which is twice that at the lower end of that part. (Each box will be supported by two pieces of elastic.) In this case that is (2 x 50) = 100g.

Stage l-Design and construction of the bridge sections

The students are supplied with card and a plentiful supply of scrap paper. They are asked to construct bridge sections that are 12 cm wide, 16 cm long and 2 cm deep. The students can be required to construct their own nets or these can be supplied printed on the card if time is short. The holes are best punched and the paper fasteners attached before the boxes are built, but the paper fasteners should not be folded too tightly for the elastic to be tied around them afterwards. The assembled bridge sections are to be filled with scrap paper, so that they have a mass of 100 grams.

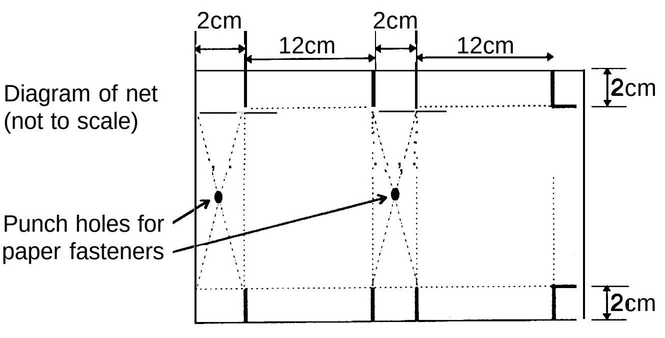

Fig. 2

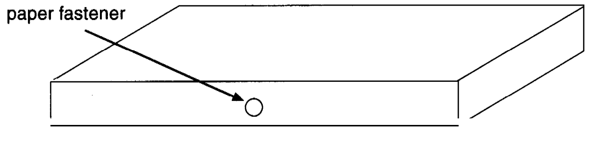

Fig. 3

Standard copier paper is 80 grams/m^2 and so it is possible to calculate how much paper is needed to give the bridge section the required mass. For the sizes specified above the area of one sheet that just fits inside the bridge section is $0.12 \times 0.16 = 0.0192$ m^2 so the number of sheets of this size required are $\frac{100}{80 \times 0.0192}$ = 65 to the nearest sheet. The box is equivalent to about 5 sheets, leaving 60 to be cut.

An alternative is to fold A4 sheets to fit into the sections. An A4 sheet is approximately 21 cm by 30 cm and so the number of complete sheets needed is $\frac{100}{80 \times 0.21 \times 0.30}$ = 20 to the nearest sheet. The box is about the same mass as two sheets, leaving 18 needed for ballast. These can easily be counted out and folded so that they fit inside the bridge section, but care is needed to keep the centre of gravity of the assembly vertically aligned with that of the box.

It is initially useful to produce about 10 sections and if the bridge turns out to be particularly successful more can be constructed as required.

Stage 2-Angles and stretch

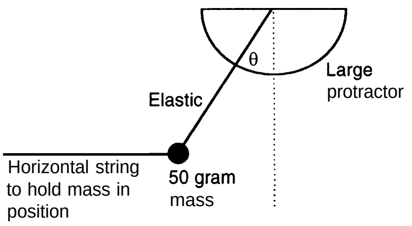

Fig. 4

This stage requires the students to carry out an experiment to determine how much a length of elastic stretches when it supports a mass equivalent to half the total (50g in this case) at various angles to the vertical. Figure 4 shows how the experiment is set up.

Make a large protractor on stiff card (radius 10-l 5 cm) with 10 degree intervals marked on it. Punch a hole through the origin and strengthen both sides with fabric reinforcements. Either attach a paper fastener through the hole or thread the end of the 35 cm piece of elastic through the hole and tie it to a paper clip. This ensures that the zero of the elastic and the origin of the protractor coincide: something the younger students found difficult. Students will need to work in pairs, one to keep the zero of the ruler at one end of the elastic and the other to read the length.

The students started with a 30 cm length of unstretched elastic and measured the stretched length for various angles θ to produce a table like the one below.

Angle	Stretched Length
0°	34.5 cm
10°	35.0 cm
20°	35.7 cm
30°	37.5 cm
40°	40.2 cm
50°	44.0 cm
60°	51.2 cm
70°	58.0 cm

A graph of the stretched length against the angle is then produced. This can be done using a spreadsheet but a hand drawn graph may equally be used. A graph based on the above data is shown in Figure 5. If the graph is plotted as the readings are being taken it is easier to spot a suspect point and re-measure it. Some year 9 students found the fact that the graph was not a straight line was disturbing, particularly as the length stays nearly constant for the first three readings. There is also a useful opportunity to discuss drawing the best curve through a set of points, since the readings rarely lie neatly on a curve. Although this may have been addressed in science lessons, students do not seem readily to transfer the skill to a different context.

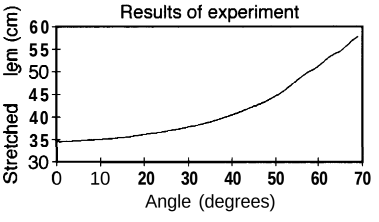

Fig. 5

The graph is then kept ready for use in the next stage of the task.

Stage 3-Cutting the elastic

The final stage is to calculate the length of elastic needed for each supporting cable. Figure 6 below shows a possible arrangement of the deck sections, elastic and retort stands, but clearly this is only one of many possible configurations.

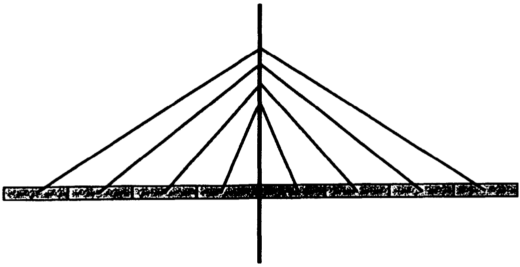

Fig. 6

The elastic is attached to the retort stands at points that are equally spaced. Using the dimensions given the students can either use scale drawing or Pythagoras and trigonometry to find the required length of elastic for each section and the angle between the elastic and the vertical.

Once this data is available the students can then find the unstretched length of elastic that will be needed to construct the bridge. For example, consider the case where the first length of elastic is attached 30 cm above the deck level and then further cables are attached at points 2 cm apart.

This requires the stretched length of the elastic to be $\sqrt{30^2 + 8.5^2}$ = **31.2** cm and be at an angle of $\tan^{-1}\left(\frac{8.5}{30}\right) = 15.8°$ to the vertical. From the graph obtained in stage 2 a 30 cm length of elastic will stretch to 35.5 cm at this angle and so to obtain a length of 3 1.2 cm when stretched an unstretched length of $\frac{312}{35.5}$ **x** 30 = 26.4 cm will be required.

Repeating this process for a number of cables leads to the type of results that are given in the table below.

Stage 4-Building the bridge

The students are now ready to attempt the construction of their bridge.

Hints For Successful Building

Students are advised to double each length of elastic, so that a single length is used to join each pair of deck sections. They are also told to add 3 cm to each length of elastic so that they can fix it to the side of the deck section.

We have found that making sleeves, like the outer part of a matchbox, is a good way to hold the bridge sections together. In an emergency, a single piece of adhesive tape along the underside of the bridge is a good way of holding a basically stable bridge together for display purposes.

It is important, the cost of elastic (14p/metre in 1997) notwithstanding, to use relatively long pieces of elastic for the first pairs of boxes. If the angle of the elastic becomes too high, the end loads in the bridge become large and a premature discovery of Euler buckling is made! The longer the bridge planned, the longer the first pieces of elastic need to be.

Horizontal distance from stand to point of attachment	Height of point of attachment above bridge level	Stretched length of elastic	Angle between elastic and vertical	Unstretched length of elastic
8.5 cm	30 cm	31.2 cm	15.8°	26.5 cm
25.5 cm	32 cm	40.9 cm	38.6°	31.5 cm
42.5 cm	34 cm	54.4 cm	51.3°	33 cm
59.5 cm	36 cm	69.5 cm	58.8°	42 cm

Accuracy in measuring and making is vital. Those students who take care right at the beginning to produce boxes with straight sharp edges often have the best bridges. The measurement of the angle and the length of the string need to be measured with great care. 'About 35.5 cm' will not do.

Teams of two or three are needed at several stages: in stage 2 two are needed to measure and a third to record, and in stage 3 many hands are needed to hold the existing boxes steady while a new pair is added.

Reactions of Students to the Task

Producing a real artefact (the bridge) is very motivating. The atmosphere in the room changes during the task. Building the boxes and ballasting them produces quiet chat. In stage 2, they are out of their seats and moving about, although the measurement and plotting need concentration. In stage 3, they are back in their seats engaged in calculations and students and the time available. In particular, the effects as the bridge gets longer are complex and interesting and can take students into mathematical modelling questions. When does the model of elastic as a linear elastic solid break down? (A practical introduction to the concept of non-linearity.) Why does the application of an end load cause the line of boxes (however carefully built) to burst out vertically? What is the longest bridge that could be made this way? How tall would the retort stand have to be? What are the factors which determine the limiting length?

What determines the angle of the elastic to each successive pair of boxes? What are the implications of varying the vertical spacing of the points of attachment to the retort stand? What factors influence the compromise between elastic angle and tower height?

Students of mechanics can be asked to draw solid body diagrams for each box, and carry out a classical mechanical analysis. The model we have used does not take into account friction between the boxes. What effect does that have?

calculator activities, and stage 4 sees the culmination of all the effort as the bridge is constructed, with success, or else needing adjustment and 'help'.

One of the outcomes of the task is that the students gain an appreciation for the importance of precision. If the boxes are not made with right angles and straight edges they will not sit square and the bridge just will not go straight.

Younger students need help and encouragement in measuring the length of the stretched elastic accurately. In particular, they tend to read the measurement end while the zero end is floating free. Reading the length from a ruler is also a good opportunity to introduce the concept of parallax.

Further Work

The bridge building activity is one which can be carried out at a variety of levels, depending on the age and ability of the

Conclusions

The task is an integrating activity which pulls together mathematical ideas such as scaling, Pythagoras' theorem and trigonometry; skills such as measuring, plotting a curve from experimental data, use of calculators, working in a group and working on a task over a period of time; and modelling ideas such as predicting the length of elastic needed and testing the prediction experimentally. This type of activity is especially valuable as many students regard knowledge as context-specific and transfer of skills is an important ability. ❑

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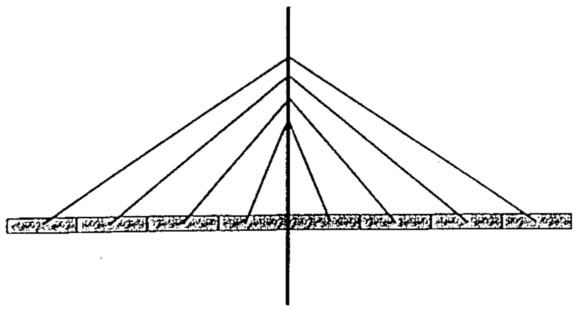
Bridge Building – Part 1

The bridge will be constructed from a horizontal deck of ballasted card boxes, suspended from a central pair of towers by means of cables (thin elastic cords).



- On A3 paper draw a scale model of the bridge. Mark the scale used on the diagram.

  Each section of the deck will be 16cm long, 12cm wide and 2cm high.
  The cable is attached 30cm above the deck level and then further cables are attached at points 2cm apart.

- Accurately measure the lengths of each cable and the angle between the cable and the tower.

- Record your results in the table below.

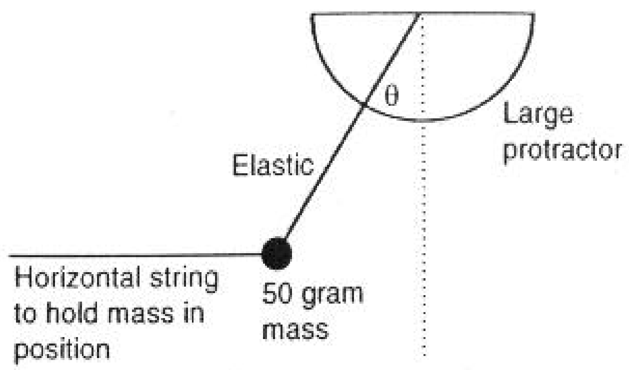
Scale Model					Real Model			
Horizontal distance from tower to point of attachment	Height of point of attachment above bridge level	Stretched length of cable	Angle between cable and tower		Horizontal distance from tower to point of attachment	Height of point of attachment above bridge level	Stretched length of cable	Angle between cable and tower

Bridge Building –Part 2

Last week we measured the stretched length of cable (elastic) we need to build the bridge. This week we will be carrying out an experiment to determine how much our cable stretches when it supports a mass equivalent to half the mass of each deck section.

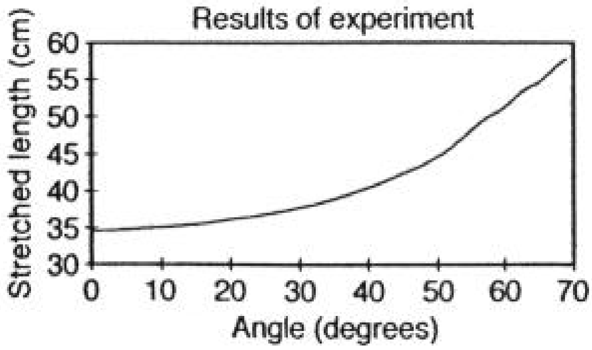
- Make a large protractor on stiff card (radius 10-15 cm) with 10 degree intervals.
- Set it up as shown below so that we can measure how much a piece of elastic 30cm in length will stretch for various angles.
- Record your results in the table.

Angles and stretch



Angle	Stretched Length

- Draw a graph of Stretched Length against Angle. Do this at the same time as you measure your lengths.



- Using the graph and the angles you measured last week, read off how much a piece of elastic 30cm long will stretch to when loaded with half the mass of a deck section.

- We now have to calculate the unstretched length of elastic we will have to cut for the cable which will stretch to the lengths we measured last week when it is loaded with a deck section.

We use the formula. $$c = \frac{a}{b} \times 30$$

Stretched length of elastic – measurement from scale diagram **(a)**	Angle - measurement from scale diagram	Stretched length of 30 cm of elastic - read off graph **(b)**		Calculated value for unstretched length of elastic (cm) **(c)**	Double the unstretched length of elastic (cm)

- We are now ready to start construction.

Bridge Building – Part 3

- Hi-ho, hi-ho it's off to work we go!!! This is the week we start construction work.
- Constructing the deck sections.
  The deck sections are 12cm wide, 16cm long and 2cm deep. They are to be made out of A4 card. (A net for a section is shown below.)

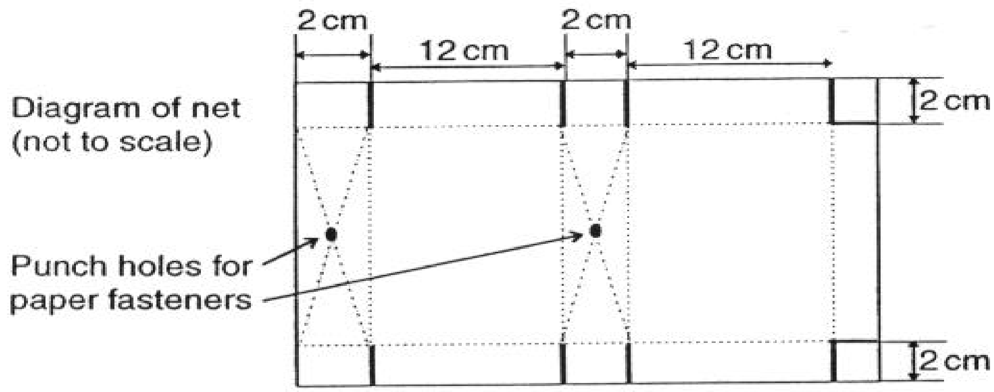
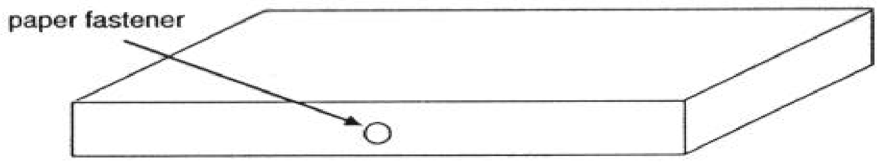


Fig. 2



Tips – Score each of the lines you need to fold, as this should give sharp edges so that sections fit snugly together. Don't seal the boxes until you have filled them with the ballast.

The deck sections have to be filled with scrap paper (60 sheets of 16cm x 12 cm paper each weighing $80g/m^2$) so their mass is 100 grams.

- Measuring and Cutting the Cables

Using the measurements you calculated in Week two cut out the lengths of elastic for the cables (a double length of elastic is used join a pair of deck sections).

(Tip – add 10cm to each length of elastic to allow for fixing to the side of the deck sections. Add a bit extra for the outer sections.)

Bridge Building – Part 4

- This is where it all comes together.
- Set up the two towers (retort stands) and clamp them together at the top so that they cannot be moved apart.
- Attach the cable supports to the towers. These are paper fasteners and it is best to attach them with sellotape.
- Put sleeves (like a match box) around adjacent deck sections to hold them firmly together. Otherwise they will spring apart at the worst possible moments.
- It is now up to you to decide how to finish of the construction.
- This is THE testing time.